

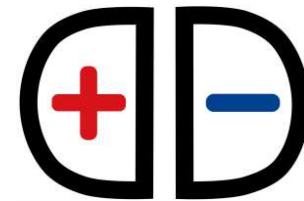
# 7. Three-Sided Dice

## 4. Unsinkable Disk

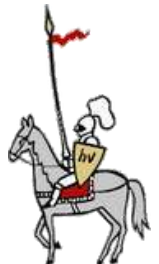
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Erasmus+ Programme  
of the European Union



## 7. Three-Sided Dice

To land a coin on its side is often associated with the idea of a rare occurrence. What should be the physical and geometrical characteristics of a cylindrical dice so that it has the same probability to land on its side and one of its faces?



## **Rebounding Capsule** (34<sup>th</sup> IYPT)

A spherical ball dropped onto a hard surface will never rebound to the release height, even if it has an initial spin. A capsule-shaped object (i.e. Tic Tac mint) on the other hand may exceed the initial height. Investigate this phenomenon.

## **Probability** (19<sup>th</sup> IYPT)

A coin is held above a horizontal surface. What initial conditions will ensure equal probability of heads and tails when the coin is dropped and has come to rest?

## **Coin** (10<sup>th</sup> IYPT)

From what height must a coin with heads up be dropped, so that the probability of landing with heads or tails up is equal?

# Simple cases

**Thin disk**



**Thin rod**



Probability of landing on any face

Probability of landing on any face

# Simple cases

**Thin disk**



Probability of landing on any face  
**100%**

**Thin rod**



Probability of landing on any face  
**0%**

# Simple cases

**Thin disk**



**Thin rod**



Probability of landing on any face

**100%**

Probability of landing on any face

**0%**

**Really always?**

# Simple cases

**Thin disk**

**Thin rod**

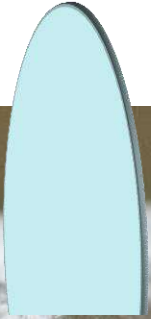


Probability of landing on any face

Probability of landing on any face

# Simple cases

**Thin disk**



**Thin rod**



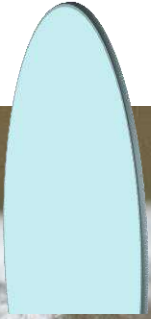
Probability of landing on any face

Probability of landing on any face



# Simple cases

**Thin disk**



**Thin rod**

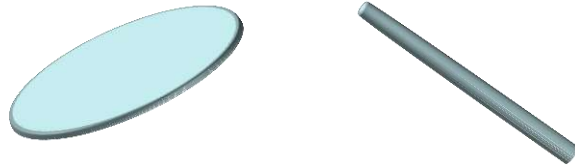


Probability of landing on any face  
**< 100%**

Probability of landing on any face  
**> 0%**

# Important parameters

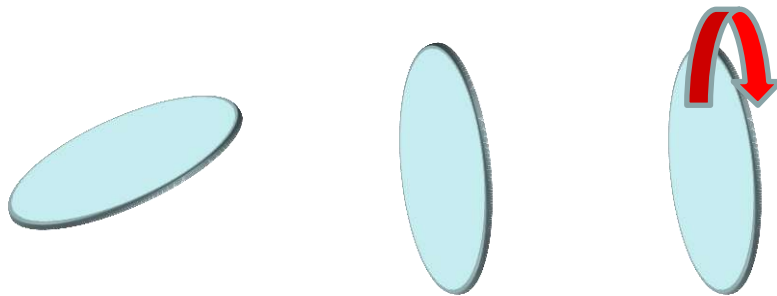
- Geometry (“the dice”)



- Target surface (“where the dice falls”)



- Initial conditions (“how the dice is thrown”)

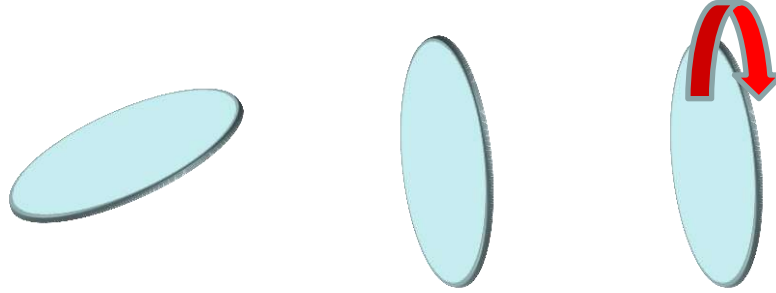


To land a coin on its side is often associated with the idea of a rare occurrence. What should be the physical and geometrical **characteristics of a cylindrical dice** so that it has the same probability to land on its side and one of its faces?

# Important parameters – initial conditions

- Initial conditions for rigid body motion → 12 parameters

- Position
- Rotation
- Velocity
- Rotation speed



- Reduction due to symmetry

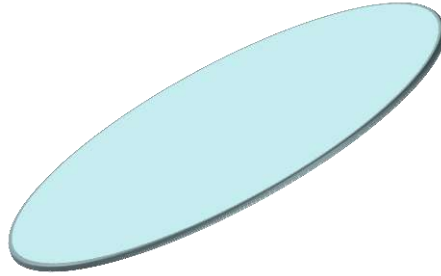
- Symmetrical cylinder in free space → 6 parameters “only” (height, lateral velocity and fall velocity, tilt, tilt speed and rotation speed along the axis)
- Cylinder with offset center of gravity → 8 parameters

# Important parameters – target surface

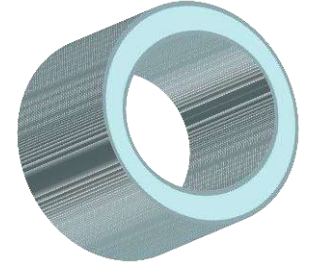
- Target surface, dice-surface interaction
  - Coefficient of restitution
  - Coefficient of friction
  - Shape of the surface
- Simplifications
  - No friction/Infinite friction
  - Coefficient of restitution  $\rightarrow 0$  (e.g. landing in flour)
- Literature: W. Goldsmith, *Impact*. London, Arnold (1960)

# Important parameters – geometry

- Aspect ratio
  - Controls the stability

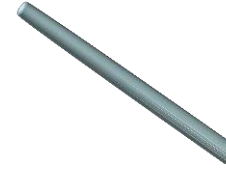
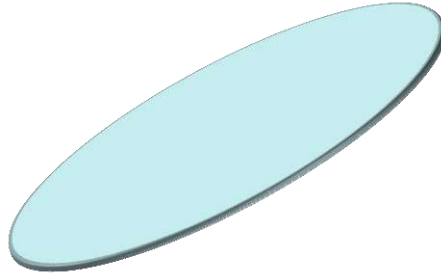


- Mass distribution
  - Controls the ratio of translation and rotation energy

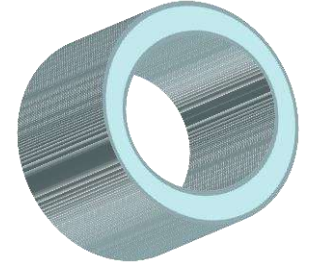


# Important parameters – geometry

- Aspect ratio
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- Mass distribution
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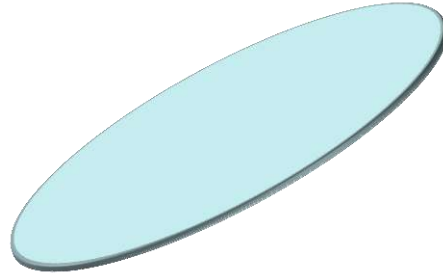


When are important the following parameters?

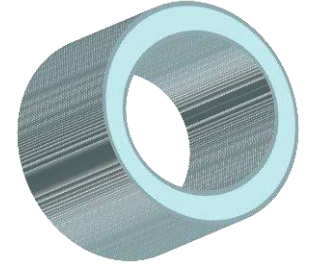
- Size
- Density of material
- (Airflow)

# Important parameters – geometry

- Aspect ratio
  - Controls the stability



- Mass distribution
  - Controls the ratio of translation and rotation energy

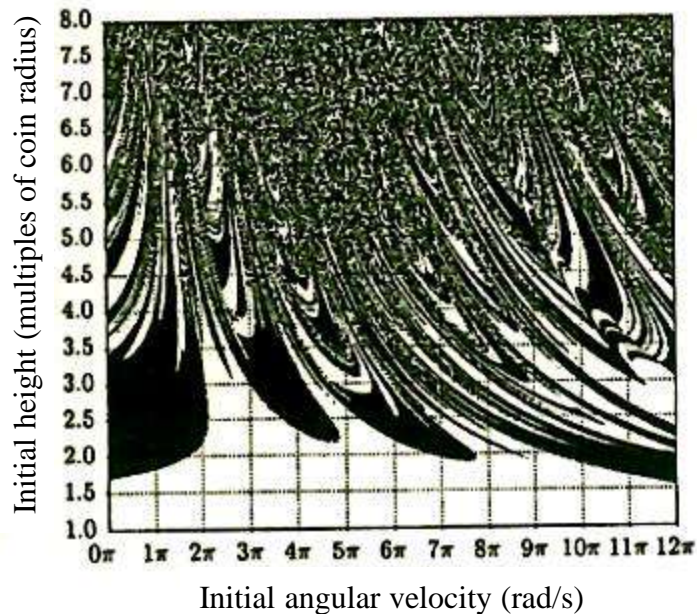


When are important the following parameters?

- Size
  - Density of material
  - (Airflow)
- } Force  $\propto$  mass
- Large speeds and/or heights

# Deterministic approach

- Initial conditions + dice geometry and physical parameters
- Dynamics of rigid body (gyroscope equation)
  - 1D: Newton's equations of translation and rotation motion
- Impact to the surface



Example: calculated diagram showing the terminal state of a 5 CZK coin on a classroom table.

White = same orientation

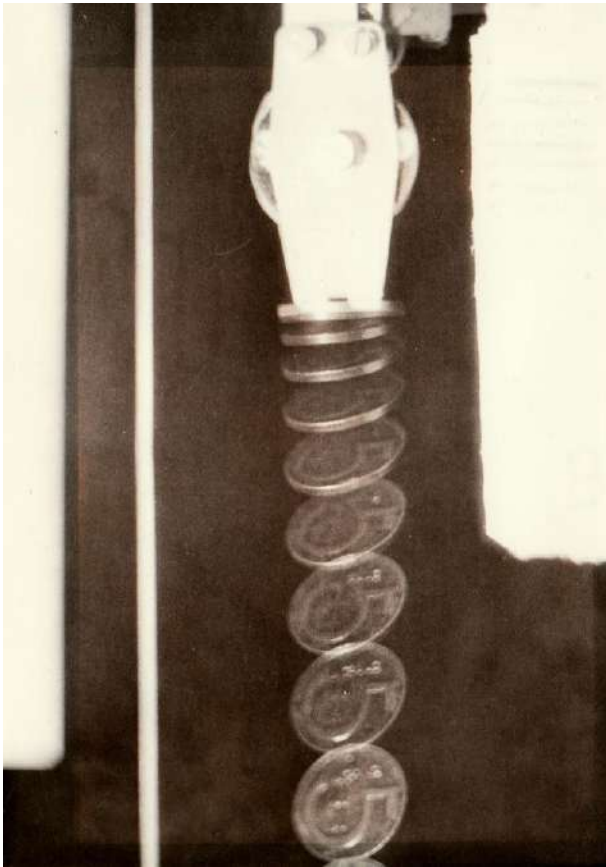
Black = opposite orientation

[In: Nápaditá fyzika, 1. vydání, ARSCII, Praha (2000), pp. 53 – 62]

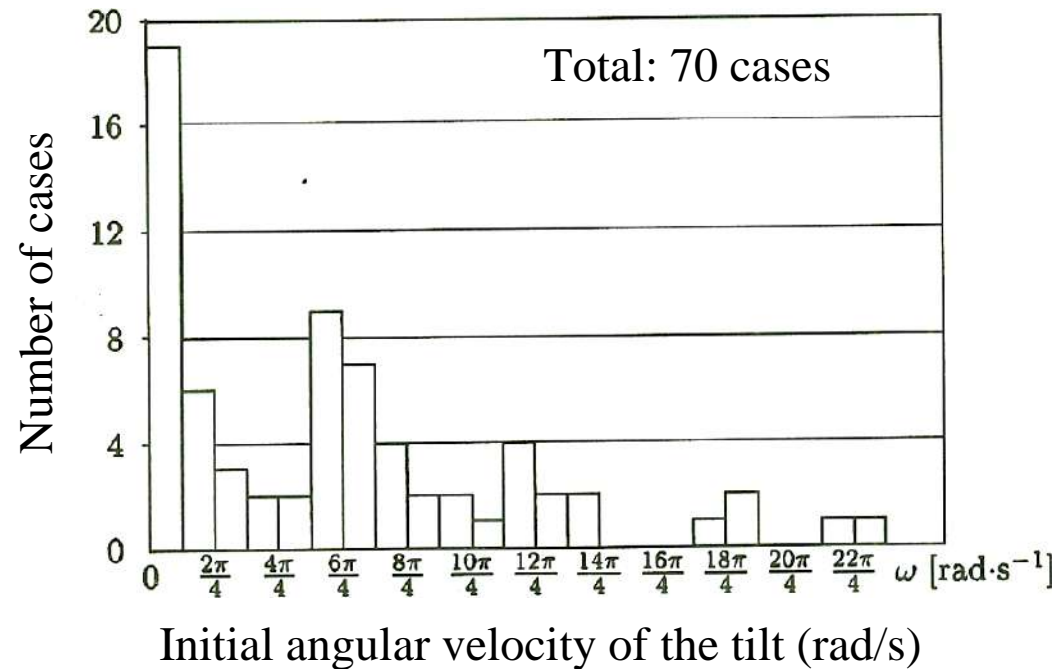


# Deterministic approach

- Issue: Control/characterization of initial conditions
  - Typically a distribution



Motion of the 5 CZK dropped by an electromagnet



# Stochastic approach

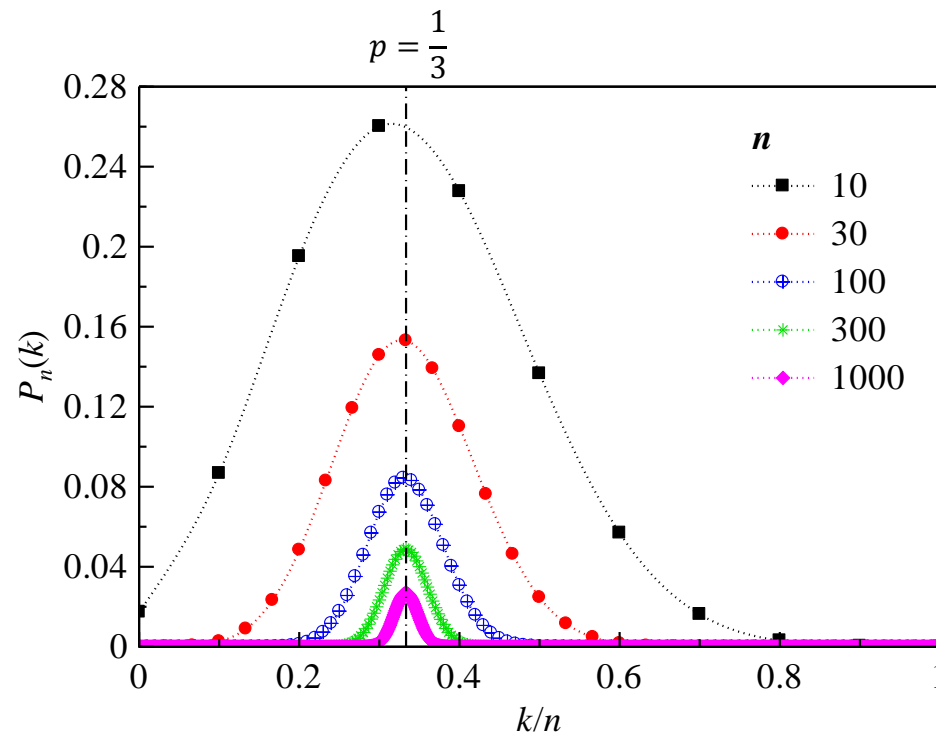
- Suitable distribution of initial conditions, e.g.
  - Initial height around 1.20 m
  - Shaking of dice assuring (quasi-)random distribution of initial tilts and a distribution of velocities
- The dice has some kinetic energy before the impact. Assess whether it may change the state upon collision
- Examples
  - Kinetic energy in the form of rotation along the main axis
  - Kinetic energy in the form of tilting
- Vary the dice shape to test your hypothesis

If the probability  $p$  of an event (e.g. cylinder landing face up) remains constant, then the probability  $P_n(k)$  that the event occurs exactly  $k$ -times out of  $n$  cases follows **binomial distribution**

$$P_n(k) = \binom{n}{k} p^k (1 - p)^{n-k} \equiv \frac{n!}{k! (n - k)!} p^k (1 - p)^{n-k}$$

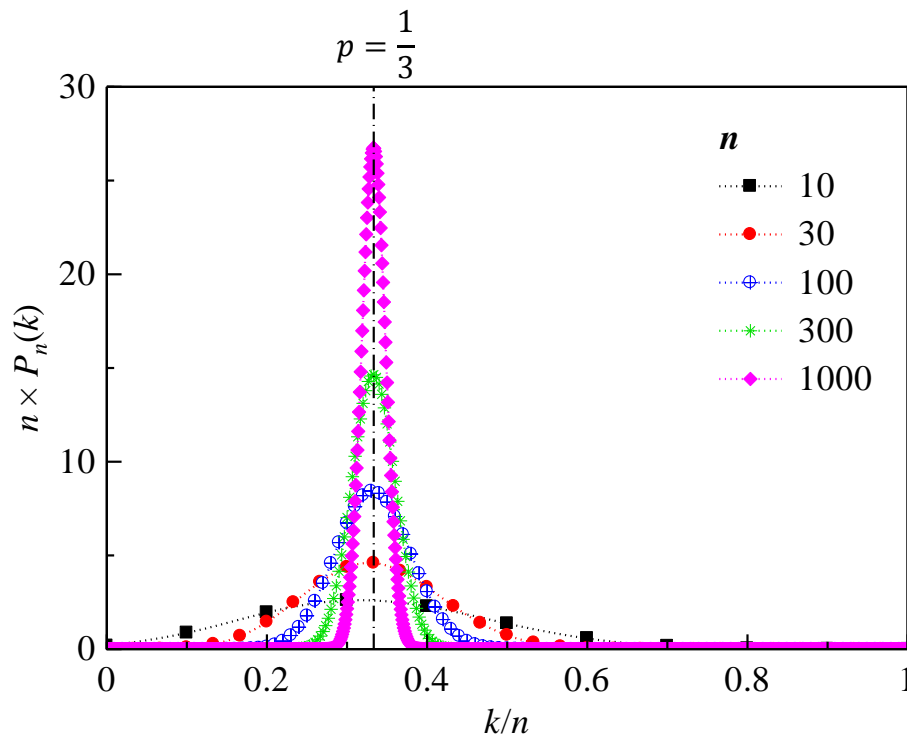
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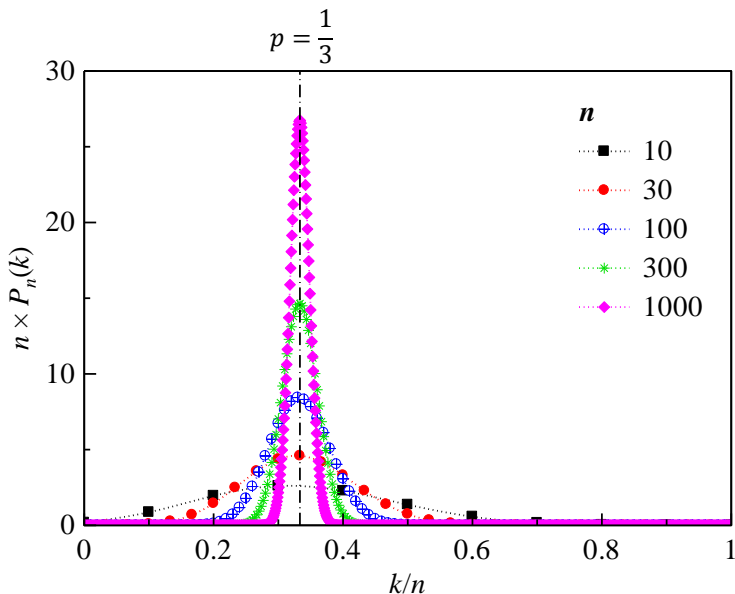
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## Important characteristics

$$\frac{\text{Mean value}}{n} = p$$

$$\frac{\text{Standard deviation}}{n} = \sqrt{\frac{p(1-p)}{n}}$$

The phenomenon happens with probability  $p$ .

What frequency shall we measure?

$n \setminus p$	1%	3%	10%	33.3%
10	$1 \pm 3$	$3 \pm 6$	$10 \pm 9$	$33 \pm 15$
30	$1.0 \pm 1.8$	$3 \pm 3$	$10 \pm 5$	$33 \pm 9$
100	$1.0 \pm 1.0$	$3.0 \pm 1.8$	$10 \pm 3$	$33 \pm 5$
300	$1.0 \pm 0.6$	$3.0 \pm 1.0$	$10.0 \pm 1.7$	$33.3 \pm 3$
1 000	$1.0 \pm 0.3$	$3.0 \pm 0.6$	$10.0 \pm 0.9$	$33.3 \pm 1.5$
3 000	$1.00 \pm 0.18$	$3.0 \pm 0.3$	$10.0 \pm 0.5$	$33.3 \pm 0.9$
10 000	$1.00 \pm 0.10$	$3.00 \pm 0.18$	$10.0 \pm 0.3$	$33.3 \pm 0.5$

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## Fundamental questions

- How many times should we throw the dice?
- How long this will take?
- When shall we become tired?



The phenomenon happens with probability  $p$ .

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**However**

( $\sigma$  denotes the standard deviation)

The result fits within  $\pm\sigma$  with 68% probability

The result fits within  $\pm 2\sigma$  with 95% probability

The result fits within  $\pm 3\sigma$  with 99.7% probability

# Statistics

The phenomenon happens with probability  $p$ .

What frequency shall we measure?

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Balance between accuracy and time: Accuracy  $\propto \sqrt{\text{time}}$

# How to start

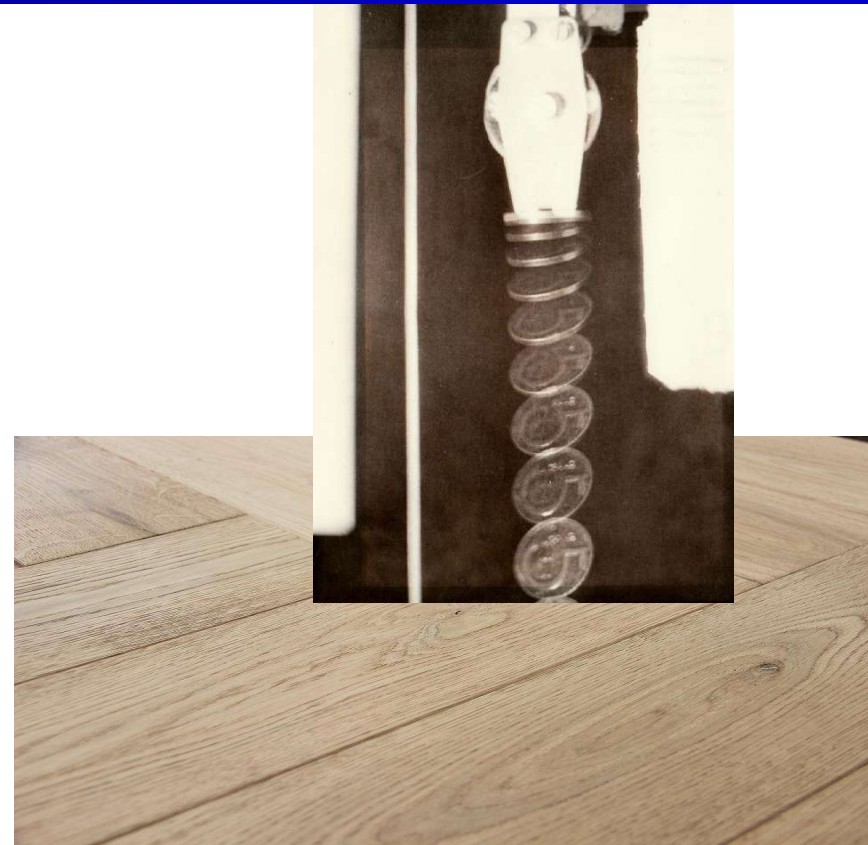
- Calculate the probability of the results for a dice in contact with the surface (consider that the orientations are random) as a function of the aspect ratio
- Perform many experiments under selected conditions. Observe whether the results agree with the simple picture:
  - Yes?  $\Rightarrow$  Explain why the complex dynamics leads to so simple observation!
  - No?  $\Rightarrow$  Explain the difference: any deviation from the simple view is interesting!

*Win-win situation!*

- Select a parameter which you vary. Ensure that the other remain as intact as possible!

# Conclusions (Three-Sided Dice)

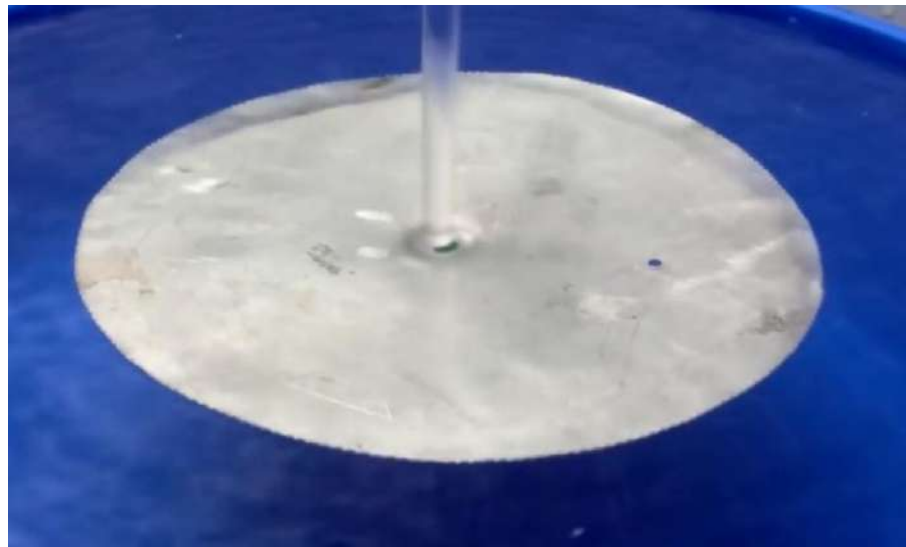
- Important parameters
  - Dice
    - Aspect ratio, mass distribution
  - Dice-surface interaction
    - Coefficient of restitution and friction
  - Surface
    - Shape
  - Initial conditions
    - Random or deterministic
- Approaches
  - Deterministic or stochastic
- Statistics
  - Doing  $< 100$  realizations makes little sense



FACE 1	FACE 2	SIDE

## 4. Unsinkable Disk

A metal disk with a hole at its centre sinks in a container filled with water. When a vertical water jet hits the centre of the disc, it may float on the water surface. Explain this phenomenon and investigate the relevant parameters.

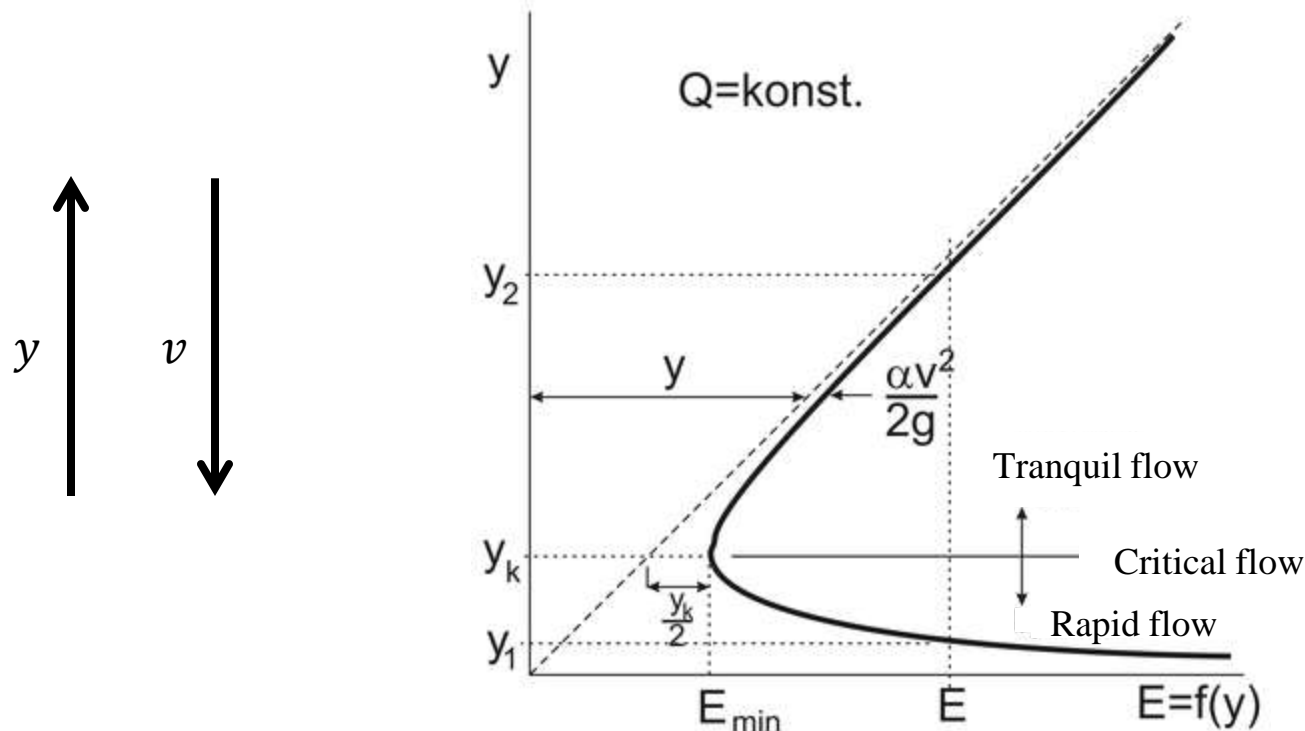


# Open-channel flow

Energy of a flow with velocity  $v$  in a open channel with depth  $y$

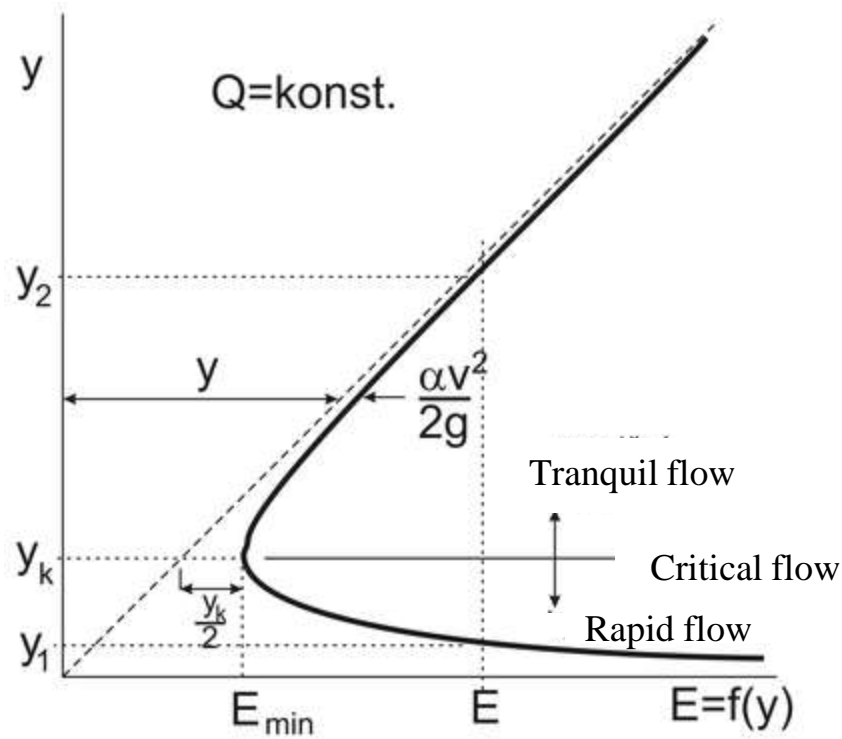
$$\frac{E}{\rho g} = y + \frac{v^2}{2g}$$

Let's assume a constant flow rate  $Q$  ( $\propto v \cdot y \Leftarrow$  mass conservation)



# Open-channel flow

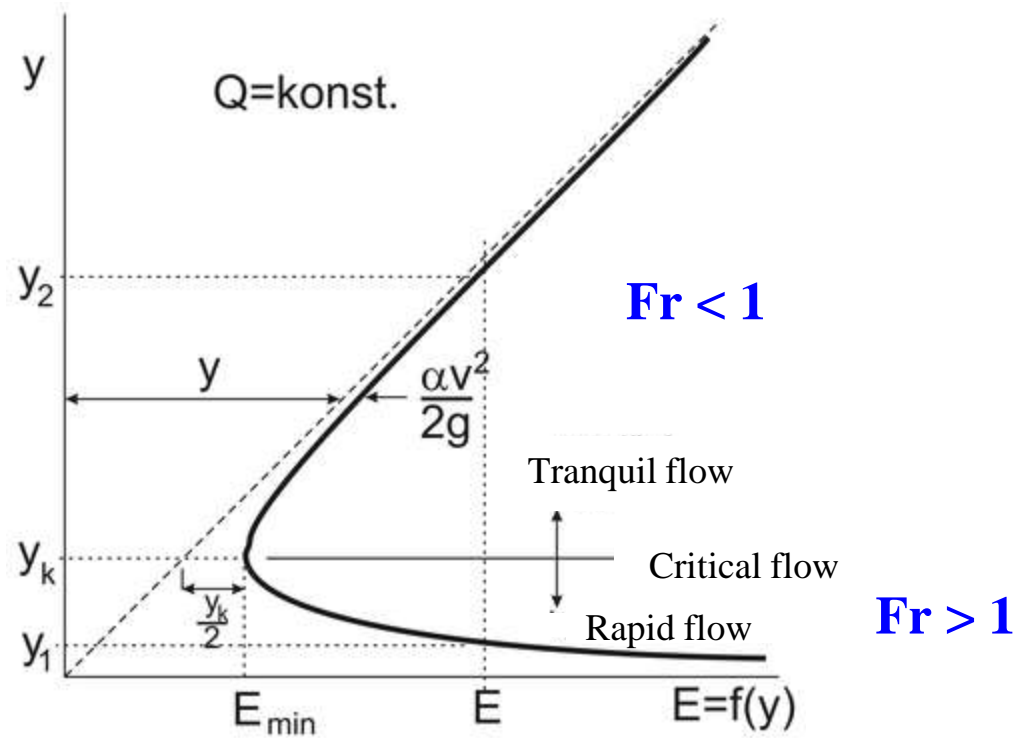
## Energy of a flow



Critical depth  $y_k$ : transition between tranquil and rapid flow

# Open-channel flow

## Energy of a flow



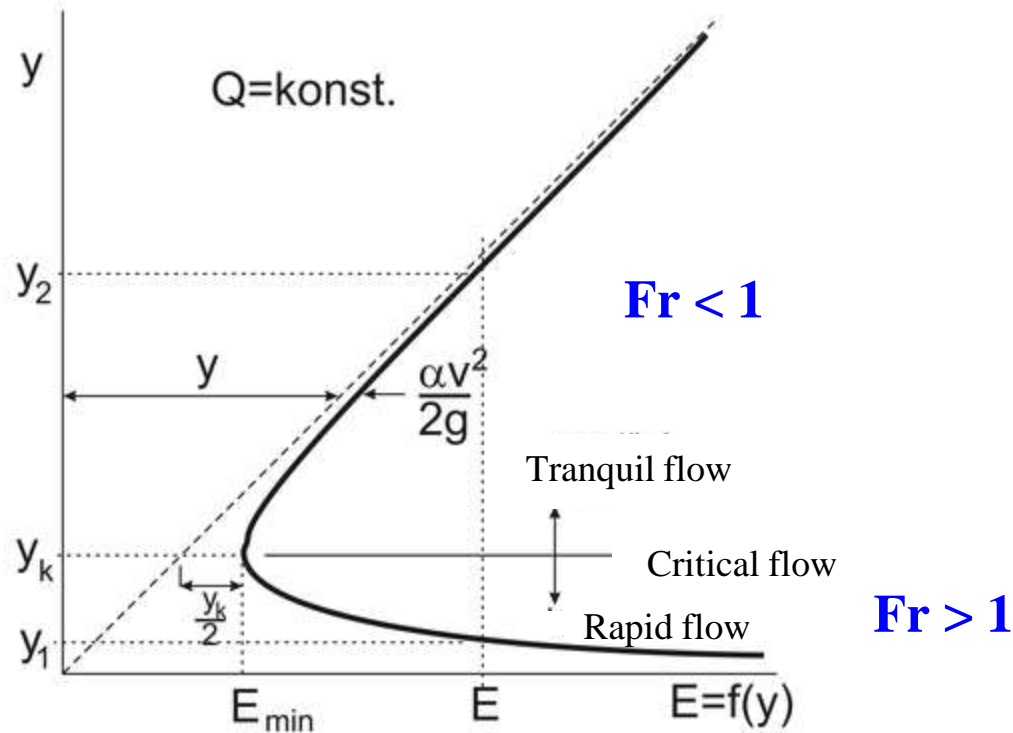
Froude number (dimensionless number in fluid dynamics)

$$Fr = \frac{v}{\sqrt{gy}} = \frac{\text{flow inertia}}{\text{gravity}}$$



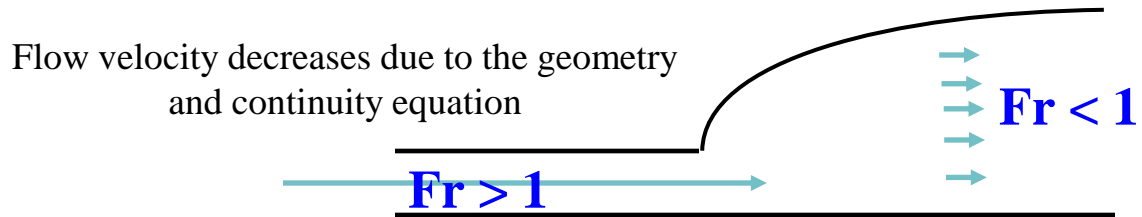
# Open-channel flow

Tranquil-rapid flow transition  $\rightarrow$  instability  $\rightarrow$  hydraulic jump



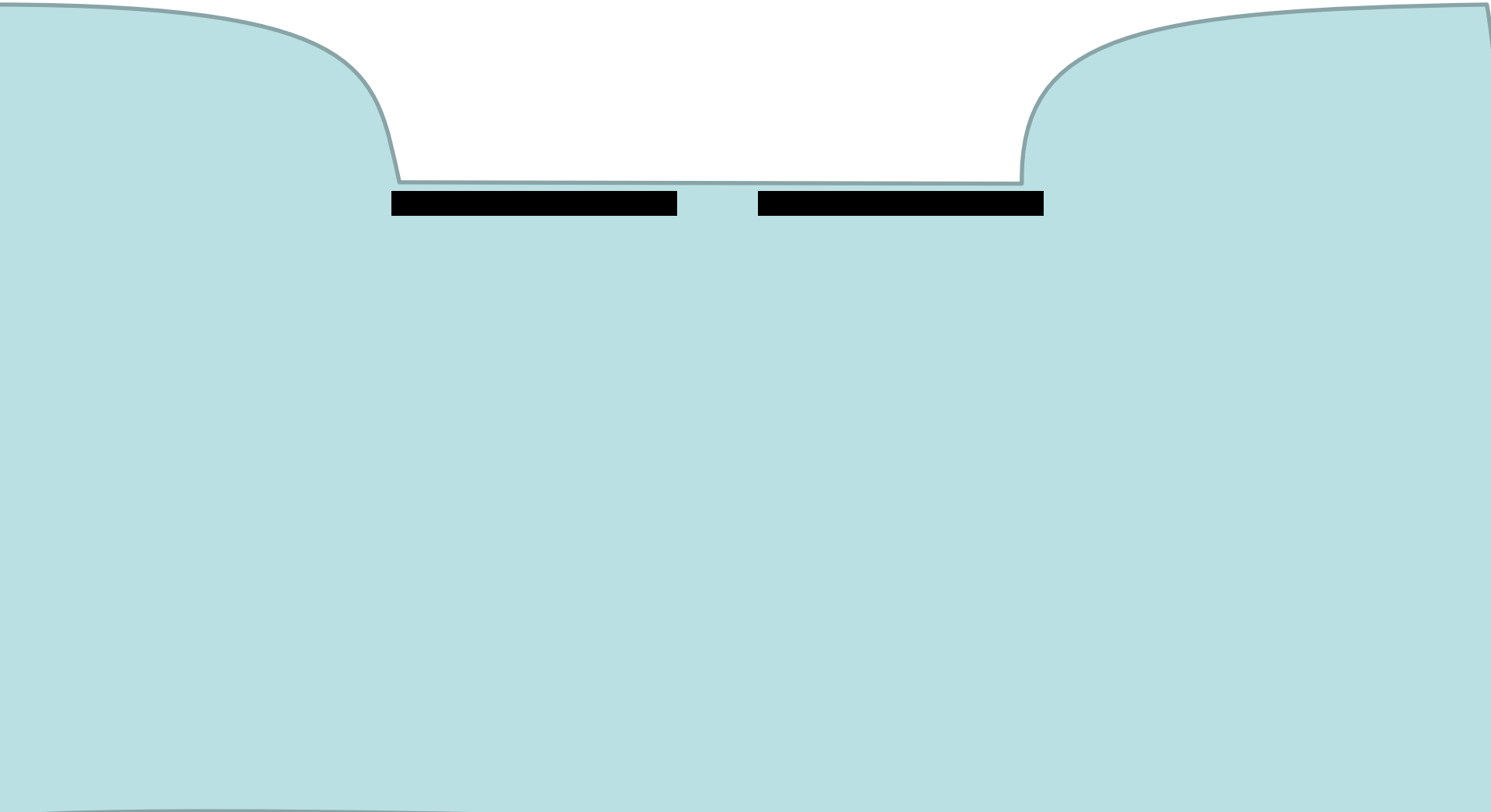
# Hydraulic jump

Tranquil-rapid flow transition  $\rightarrow$  instability  $\rightarrow$  [hydraulic jump](#)



# Buoyancy

Archimedes law



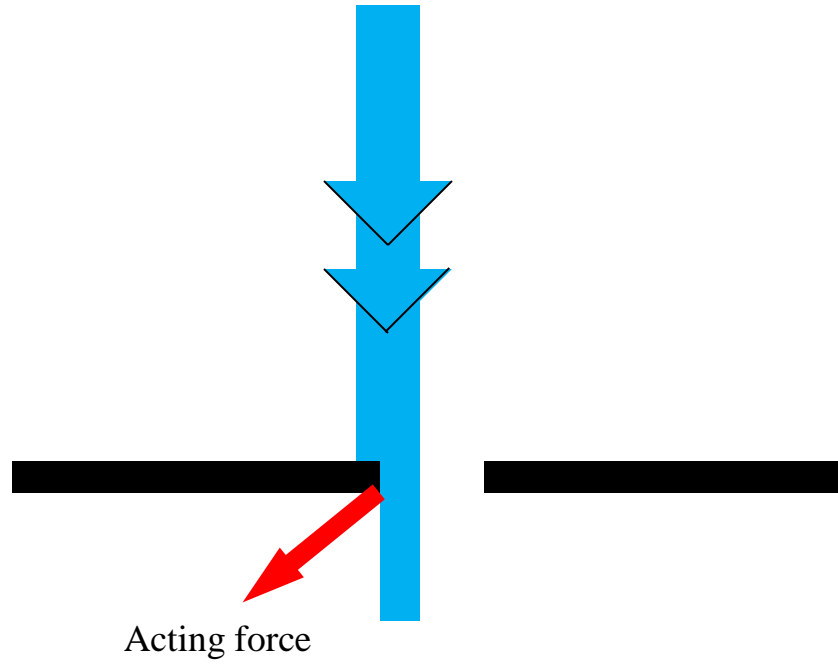
# Buoyancy

Archimedes law



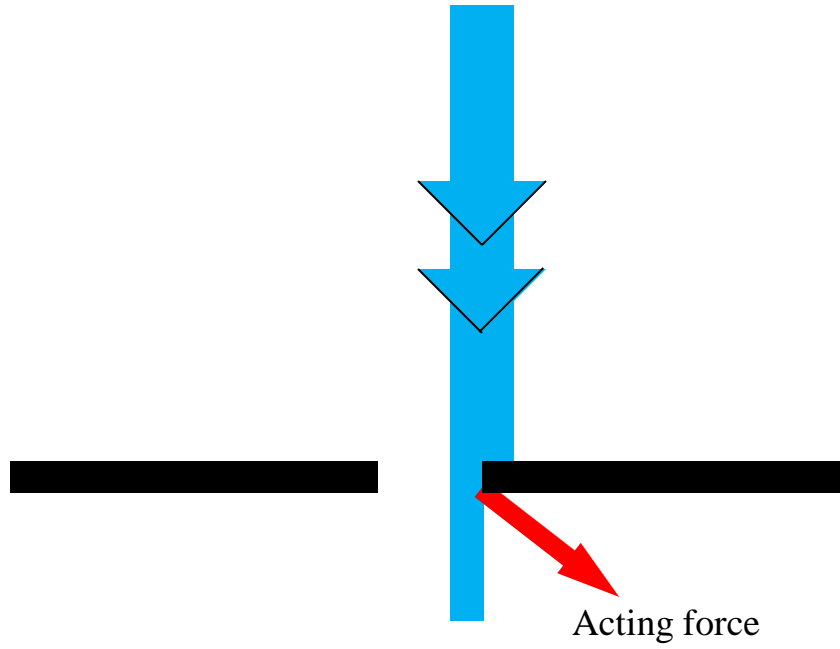
# Stabilization

Restoring force



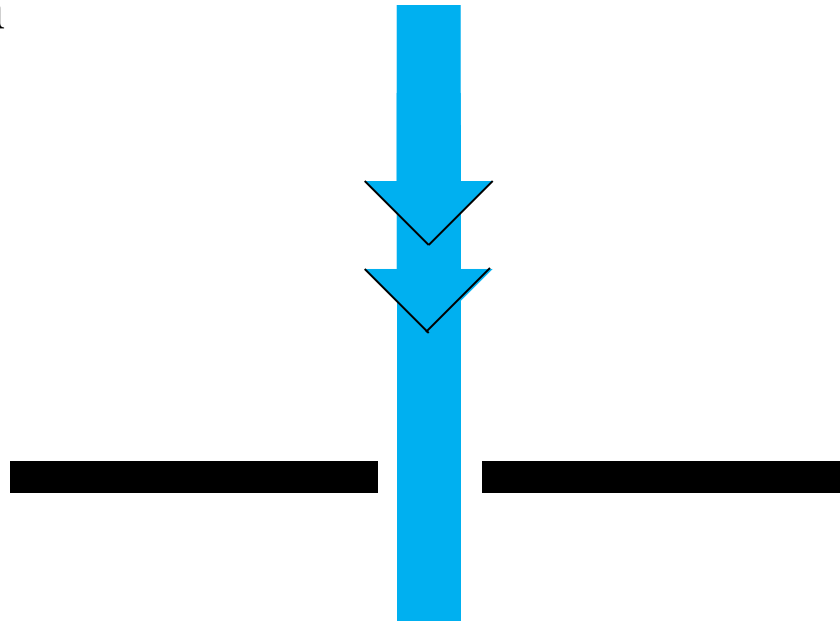
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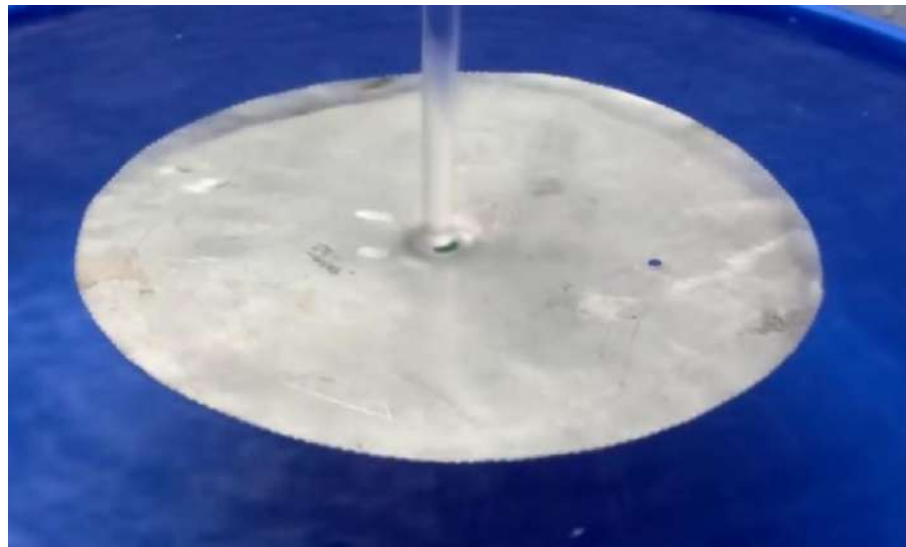
# Stabilization

Optimal position



# Conclusions (Unsinkable Disk)

- Hydraulic jump
  - Expels water from the space above the disc
- Buoyancy force
  - Pushes the disc up
- Restoring force
  - Prevents the disc from escaping





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Plenty of effects  
worth investigation

