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IYPT
International Young Physicists' Tournament

17. Hand Helicopter

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A simple hand helicopter can be made by attaching rotor blades to one end of a vertical stick. The helicopter moves upwards when the stick is twisted at a high enough speed and then let go. Investigate how the relevant parameters affect the lift-off and the maximum height.

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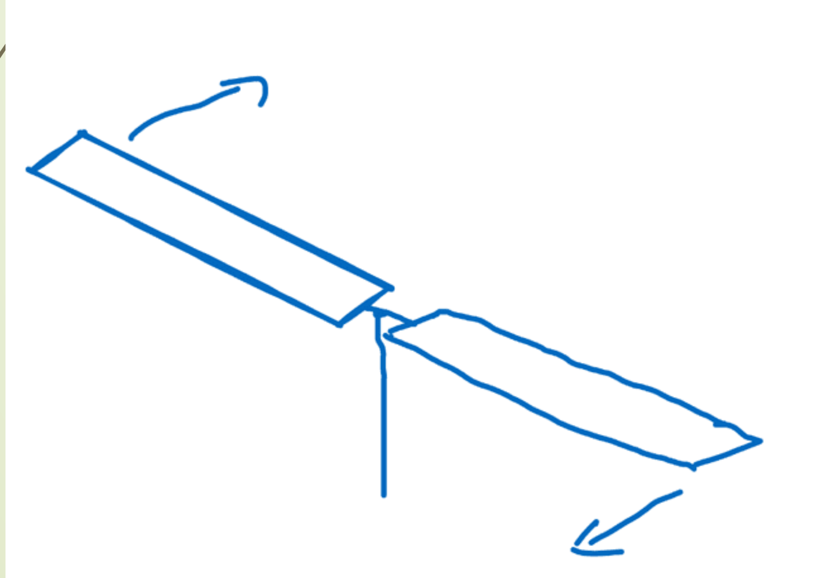
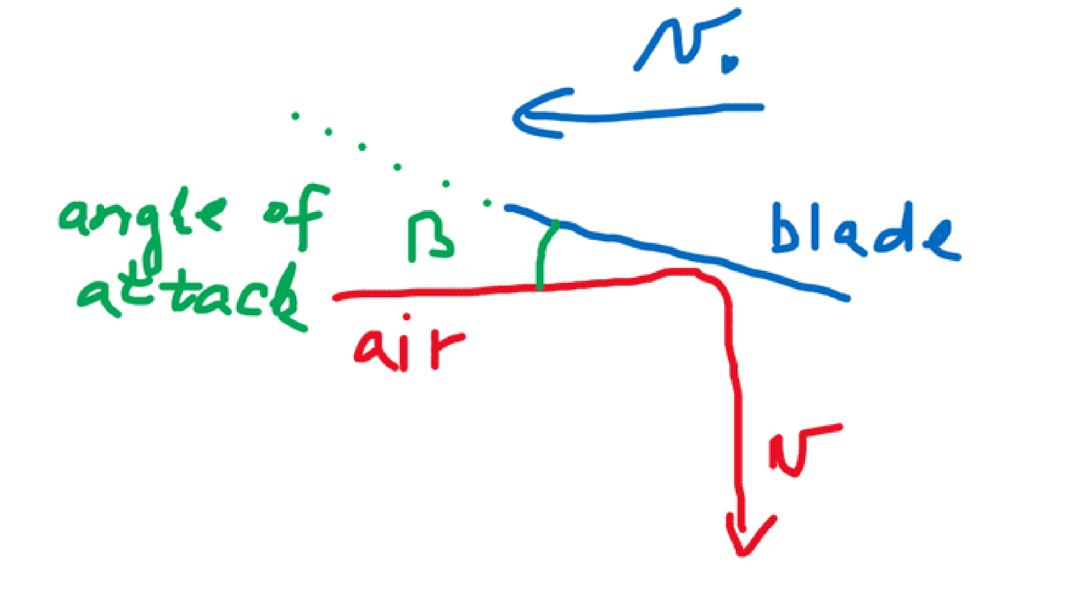
Thrust force

- Air is pushed down due to angle of attack β
- Air vertical velocity – estimated as lossless rebound of a ball

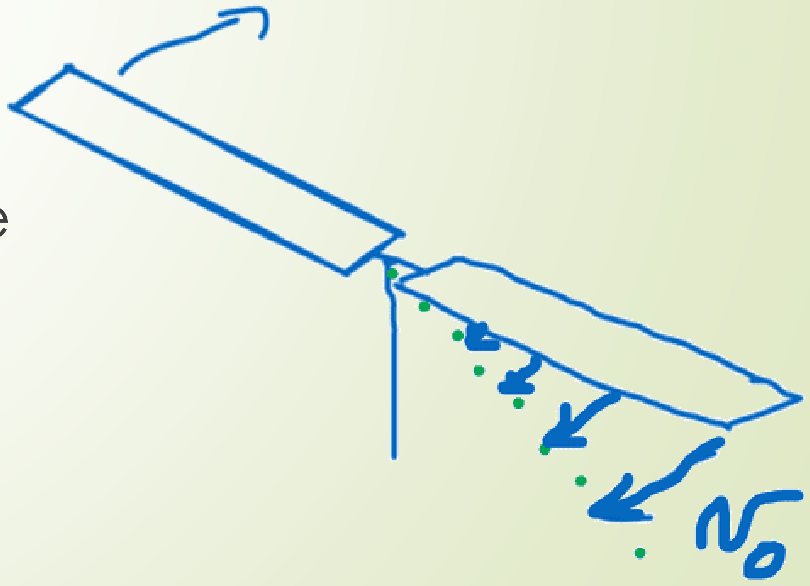
$$v \approx v_0 sin2\beta$$





Thrust force

- Attention!
  - Velocity of the blade changes along the blade! An effective velocity (smaller than the velocity of the end of the blade) has to be used!
  - The effective force acting on the air depends also on the number of blades and their area!
    Air is a continuos media, so some free space between the blades is acceptable, but you have to increase the number of blades, if they are thin!



Thrust force

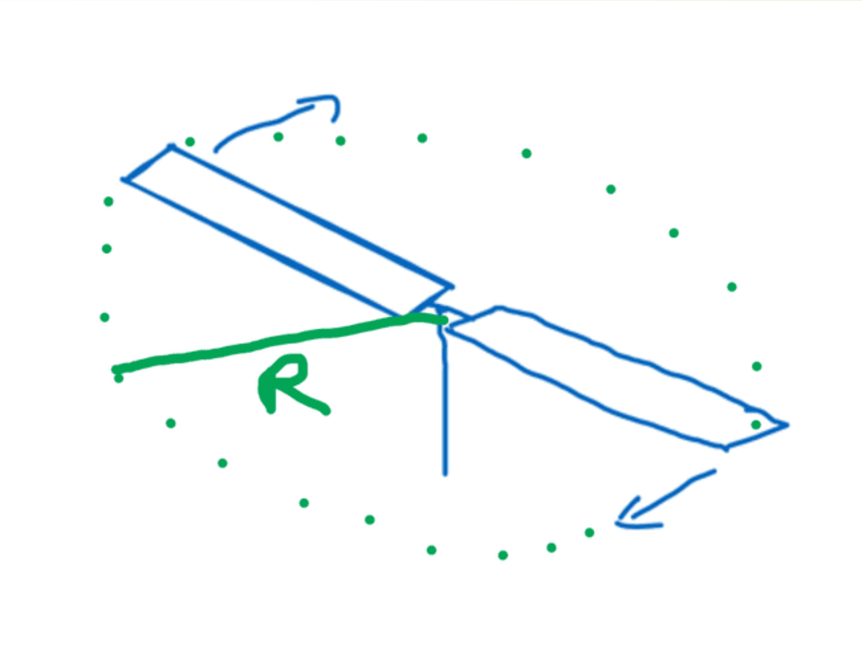
- Volumetric flow of the air (litres per second) is:

$$\frac{V}{t} = vS$$

S - circular area covered by rotating blades

$$S = \pi R^2$$

R – length of the blades



Thrust force

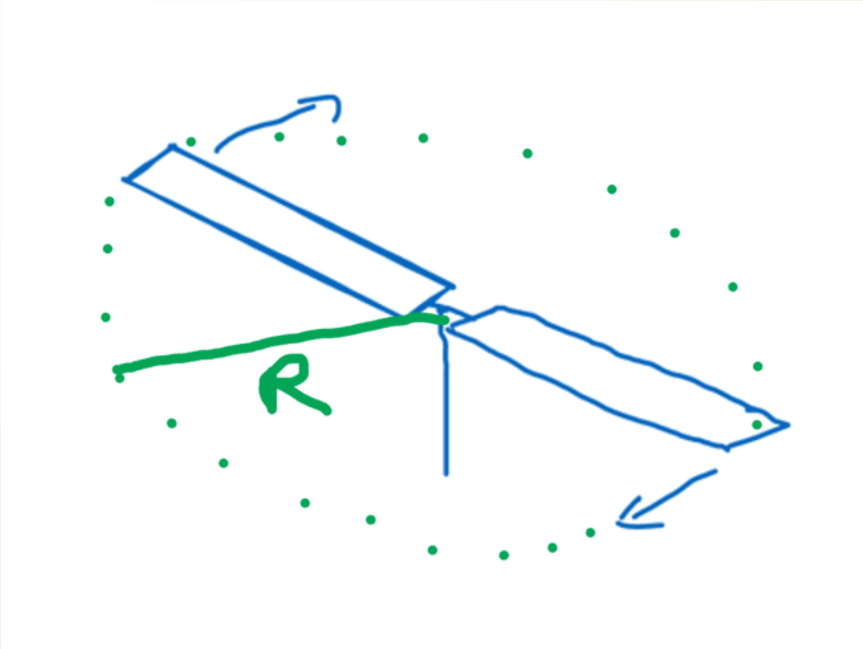
- mass flow of the air (kilograms per second) is:

$$\frac{m_{air}}{t} = \rho v S$$

ρ – density of the air

- This air must get the momentum

$$\frac{momentum}{time} = v.\rho v S$$



Thrust force

- The force acting on the air is the same as the lift-force acting on the blades

$$F_L \approx \rho v^2 S$$

The lift-force is usually expressed in the form

$$F_L = C_L \frac{1}{2} \rho v^2 S$$

C_L – dimensionless constant, $0 < C_L < 1$

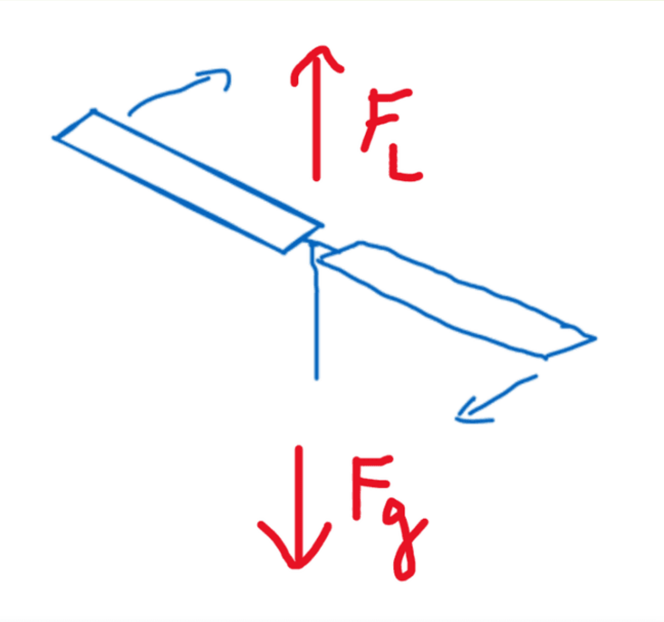
$$F_L = C_L \frac{1}{2} \rho S v_0^2 . sin^2 2\beta = C_L \frac{1}{2} (\pi R^2)(2\pi R f)^2 sin^2 2\beta$$

f – rotational frequency of the blades in Hz

Thrust force

- Finally

$$F_L = C_L 2\pi^3 R^4 f^2 \sin^2 2\beta$$



- The lift-force shoud overcome the gravitational force acting on the whole mechanism

$$F_L > mg$$

m - mass of the toy, g – gravitational acceleration

$$f > \frac{1}{R^2 \sin 2\beta} \sqrt{\frac{mg}{C_L 2\pi^3}}$$

Lift-up condition

$$f > \frac{1}{R^2 \sin 2\beta} \sqrt{\frac{mg}{C_L 2\pi^3}}$$

- The nominator should be as low as possible
  - Low mass m of the toy
- The denominator should be as big as possible
  - Optimal angle of the attack is ca 45°
  - Scaling the toy is not so much effective (mass ~ R^3)

$$f_{min} \sim \frac{\sqrt{R^3}}{R^2} = \frac{1}{\sqrt{R}}$$

Lift-up condition

- Estimation of the minimum frequency:
  - R=0.01m (blades 10 cm)
  - m=0.01kg
  - C_L=0.2

$$f_{min} \sim \frac{1}{0.1^2 \times 1} \sqrt{\frac{0.01 \times 10}{0.1 \times 2 \times 3.14^3}} \sim 12\mathrm{Hz}$$

- A realistic result, C_L was only estimated, no idea about its real value

Flight time/height

- The toy will fly up until the rotational frequency is high enough (the lift-up condition must be met)
- The rotation is slowed down due to
  - Accelerating the air – usually the main effect
  - Drag force acting against the moving blades – can be dominant at high angles of attack – optimal angle of the attack should be less than 45 degrees, perhaps 10-30 degrees?
- Power needed for the acceleration of the air (in watts)

$$P = F_L v = C_L \frac{1}{2} \rho v^2 S . v = C_L \frac{1}{2} \rho v^3 S$$

$$P = 4 C_L \pi^4 R^5 f^3 \sin^3 2\beta$$

Flight time/height

- The power needed comes from the rotational energy of the toy

$$W = \frac{1}{2} I\omega^2 = 2\pi^2 I f^2$$

- The rotation is slowing down due to power given to the air

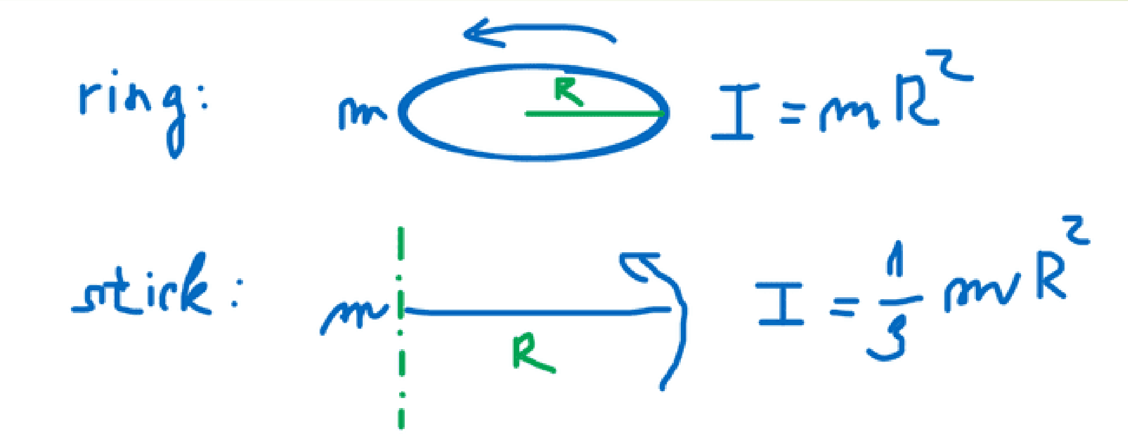
$$\mathrm{P} = \frac{\Delta W}{\Delta t} = \frac{\Delta\left(\frac{1}{2} I\omega^2\right)}{\Delta t} = \mathrm{I}\omega \frac{\Delta \omega}{\Delta t} = 4\pi^2 I f \frac{\Delta f}{\Delta t}$$

- I – moment of inertia

Generally:

$$I = C_I m R^2$$

C_L – constant $0 < C_L < 1$



Flight time/height

- How long takes the slowing down?

$$\Delta t = 4\pi^2 I f \frac{\Delta f}{P}$$

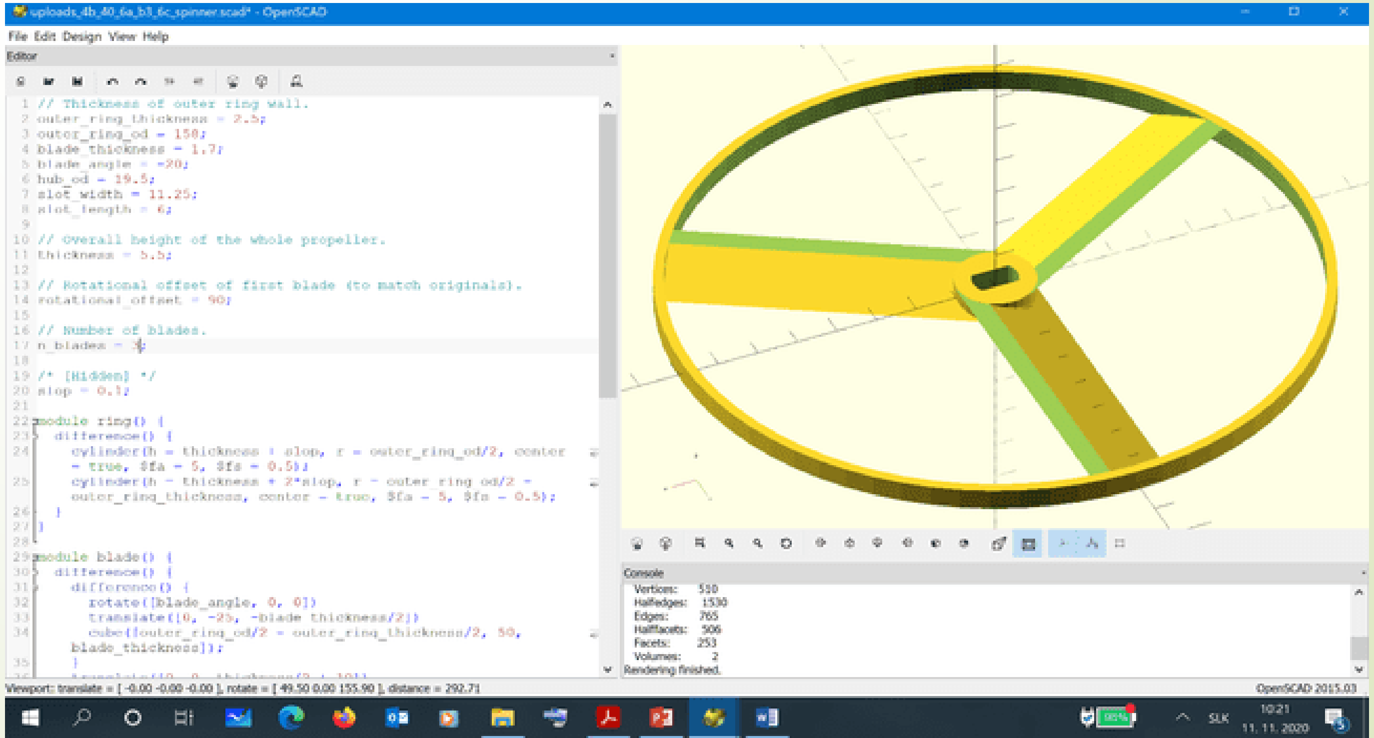
$$\Delta t = \frac{C_I m \Delta f}{C_L \pi^2 R^3 f^2 \sin^3 2\beta}$$

- What can we see:
  - Scaling is not very effective, $m \sim R^3$,
    Δt doesn't depend on R
- Estimated time of flight
  $C_L = 0.5$
  $\Delta f = 0.5f$ (toy gets 50% more rotation at the begin than is the minimal one)

$$\Delta t = 3s$$

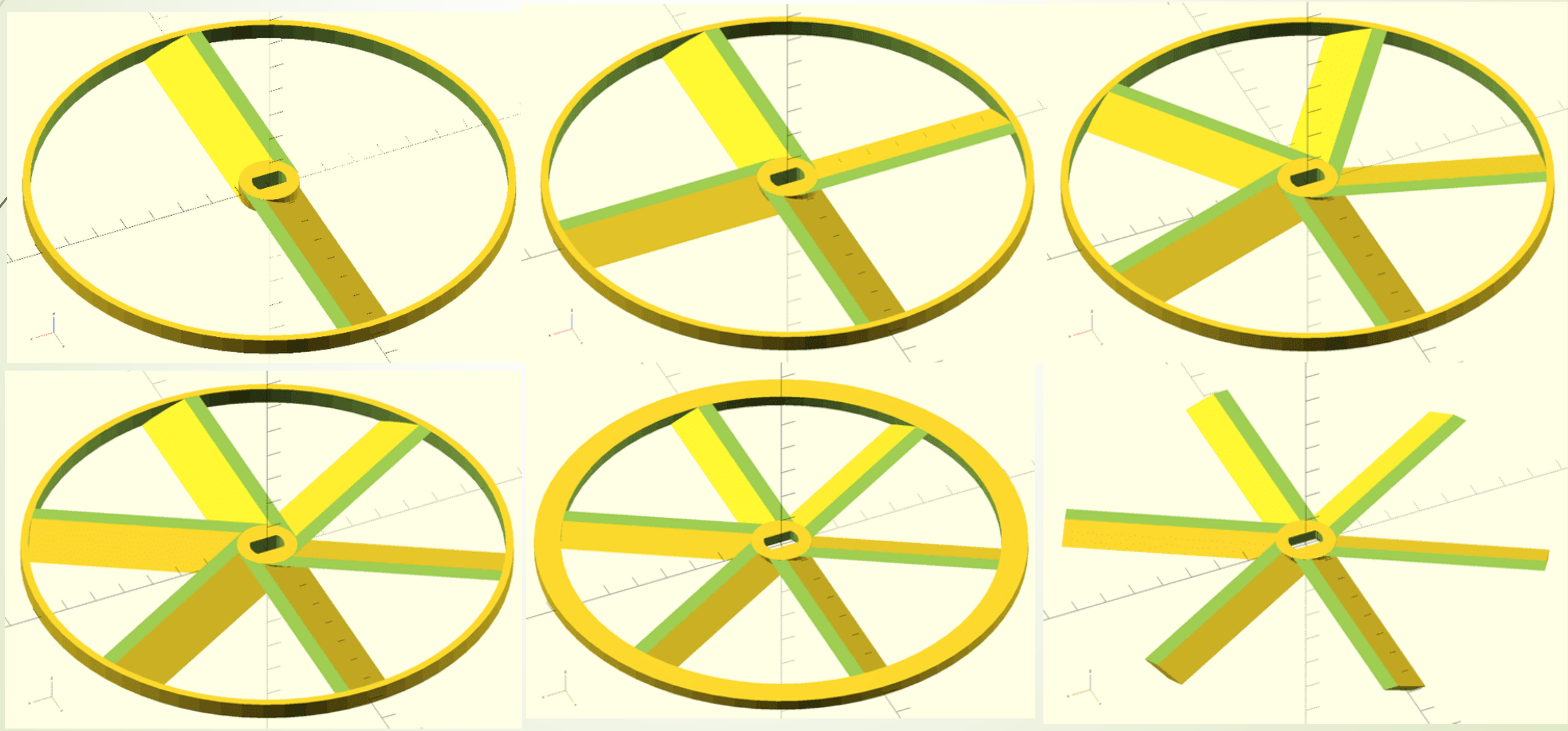
What to do?

- Prepare a set of toys – various parameters
- 3D printing is a good possibility
- https://www.thingiverse.com/thing:291404
  - A simple customizable SCAD model, good starting point to own design



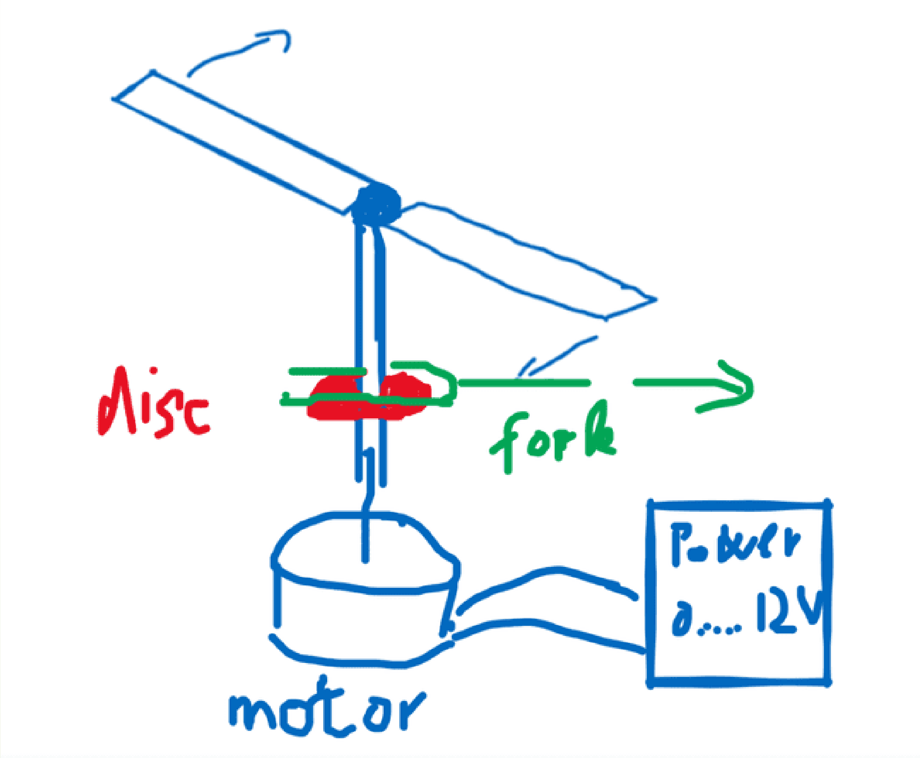
What to do?

- You can easily change: number of blades, width of blades, angle of attack, mass of the ring, scaling, ...



What to do?

- Build a motor driven starting device
- Frequency controlled by voltage source
- Frequency measured by a LED-stroboscope, for example, search Google Play for App like „Strobily“
- Releasing – remove the fork



What to do?

- Measure the lift-force using kitchen weights, for example
- Compare the lift-up force with the theory

What to do?

- Measure the slow-down of the rotation
- Optimize the toy for maximum lift-force and minimum slow-down
- Measure the flight-high for various designs, find maximum
- Critically compare results with the theory – the theory is simplified, lift-up coefficient and/or drag coefficient can depend on the frequency, for example (they were epected to be constants)

Literature

- http://sphsdevilphysics.weebly.com/uploads/5/0/7/1/5071691/example02_en.pdf
- https://www.coloradocollege.edu/academics/dept/physics/images/posters_18_19/Harm_Poster.pdf
- https://www.intechopen.com/books/flight-physics-models-techniques-and-technologies/helicopter-flight-physics
- And many others...

Thank you for your attention