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10. Nejasný smer

Ak rozkotúľame prstenec v parabolickej miske, môže sa začať zaujímavo pohybovať. Preskúmajte tento jav.

10. Spin Drift

When a ring is set to roll in a parabolic bowl, interesting motion patterns may arise. Investigate this phenomenon.

Showcase

- www.instagram.com/p/B8cGD6tBoFU/
- www.youtube.com/watch?v=Eb-9w2AMvpw

Essence of the problem

- Basically purely mechanical problem
- **Approximations** to consider
  - Ring always touches the round
  - Ring does not slip
  - Ring does not crash to the ground
    - Coefficient of restitution
  - The bowl is large enough

Position of the ring

- Position of the center of mass
  - **x, y, z**
  - **R, theta, z**
- Orientation of the ring (riadus **r**)
  - **Phi** and **psi**
  - Third angle irrelevant

- One boundary condition – touching the ground
  - Condition on **z**, highly nontrivial

Movement of the ring

- Speed of the center of mass
  - $\mathbf{v_x}$, $\mathbf{v_y}$, $\mathbf{v_z}$
  - $\dot{R}$, $\dot{\phi}$, $\mathbf{v_z}$
- Orientation of the ring
  - $\dot{\varphi}$ and $\dot{\psi}$
  - The rotational speed is relevant here, ω
- Four boundary conditions
  - Touching the surface of the bowl (two)
  - No-slipping (two)

Energy of the ring

- Kinetic energy (mass of the ring **m**)

$$E_k = \tfrac{1}{2} m\left(v_x{}^2 + v_y{}^2 + v_z{}^2\right) = \tfrac{1}{2} m v_z{}^2 + \tfrac{1}{2} m\dot{R}^2 + \tfrac{1}{2} m R^2 \dot{\phi}^2$$

- Potential energy

$$E_p = mgz$$

- Rotational energy

$$E_{rot} = \tfrac{1}{2} m R^2 \left(\omega^2 + \frac{2}{\pi}\left(\dot{\psi}^2 + \dot{\varphi}^2\right)\right)$$

Solution

- Impossible in the simple energy / force picture
- No approximations seem to make any sense
  - Except of flat surface
  - That seems to be to far away from the task statement

- All or nothing type of problem
- Only almost full solution possible

Full solution (no slip)

- Lagrangian

$$L = T - V = E_{kin} - E_{pot}$$

- Set of Lagrange equeations

$$\frac{\partial L}{\partial q} - \frac{\mathbf{d}}{\mathbf{d}t}\frac{\partial L}{\partial \dot{q}} + \sum_{i=1}^{5} \lambda_i \frac{\partial f_i}{\partial q} = 0$$

- **q** are coordinates and **f** boundary conditions
- You do not need to solve the boundary conditions separately

Full solution (slip)

- Numerical simulation
- Take care about aggregation of imperfections

Experiment

- Hard surface and ring
  - Steel
  - Ceramics?
- Narrow and thin ring
  - Rounded edge, if possible
- High speed recording of the movement
  - More than one camera, if possible
  - Color marks on the ring

What to do - Minimum

- Prepare a reasonable experiment
- Find a reasonably stable way of starting it
- Find some reproducible, yet interesting paths of the ring

What to do – higher level

- Simulate the problem

  or

- Put down the Lagrange equations and numerically solve them
- Relate at least qualitatively these results to your experiment

What to do – IYPT champions

- High quality experiment
  - Automated tracking system allowing data gathering
  - Low energy loss
  - Reproducible starting
- Account for loss of energy
  - Rolling friction
  - Slip
  - Crash – coefficient of restitution
- **Quantitative agreement between theory and experiment**